

# Math 518

## Final Exam Instructions

Monday, June 16, 2014

Room \_\_\_\_\_

12:30 – 2:15 pm

- Bring your **textbook** to the final exam. If you lost your textbook, please bring cash or a check payable to City of Newton for the book's cost. Algebra 2, (Schultz), \$70.25.
- Bring your own calculator to the exam. **I do not have any to lend out.** Sharing calculators or using someone else's when they are finished is **not** allowed. Calculator memories will be cleared before and after the exam.
- Don't forget your **notecard!**
- The exam will start promptly at 12:30 pm and end promptly at 2:15 pm. (No extra time allowed, unless you have an IEP that specifies extra time. If you have an IEP or 504 plan that allows for extended time, I will assume you will **stay after the exam** and continue working. If that will be a problem, please see me IN ADVANCE.)
- You may NOT leave the room during the exam, including to go to the bathroom.
- No electronic equipment (iPods, mp3 players, etc.) may be out during the exam. Bring a watch to keep time during the exam. Your cellphone must be off and turned in at the beginning of class or you may choose not to bring it to class that day.
- If you are legitimately absent the day of the exam, your parents must call the house office and you may make up the exam on make-up day, Thursday, June 19 at \_\_\_\_\_ am, or you will have to arrange to make up the exam in September.
- Take a deep breath and **relax!**

<b>Final Exam:      Monday, June 16, 2014</b> <b>                         12:30 – 2:15 pm</b>
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<b>Be sure to bring: pencil, <i>calculator</i>, notecard, <i>textbook</i></b>
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**By the end of this course you should be able to:**

**1. Sequences and Series (Sections 11.1 – 11.5 in Algebra 2)**

- Find the terms of a sequence given an explicit or recursive formula
- Evaluate the sum of a series expressed in sigma notation
- Recognize arithmetic sequences
- Find the indicated term of an arithmetic sequence
- Write and use explicit and recursive formulas for arithmetic sequences
- Find arithmetic means between two numbers
- Find the sum of the first  $n$  terms of an arithmetic series
- Use the formula for an arithmetic series
- Recognize geometric sequences
- Find the indicated term of a geometric sequence
- Find geometric means between two numbers
- Find the sum of the first  $n$  terms of a geometric series
- Use the formula for a geometric series

**2. Exponents (Section 2.2 in Algebra 2)**

- use properties of exponents to simplify expressions
- evaluate and simplify expressions with negative exponents
- evaluate expressions with fractional exponents
- rewrite expressions with fractional exponents using radicals and vice versa

**3. Functions (Sections 2.3, 2.4, 2.5 in Algebra 2)**

- tell whether a relation is a function
- define a function
- state the domain and range of a given relation
- write a function in function notation and evaluate it
- interpret statements written in function notation
- perform operations with functions (add, subtract, multiply, and divide) to write new functions
- identify any domain restrictions when dividing functions
- find the composite of two functions
- find the inverse of a relation or function given a graph, equation, or set of points
- determine whether the inverse of a function is a function (whether the function is one-to-one)

#### 4. Quadratics

(Sections 5.1, 5.2, 5.3, 5.4, 5.5 in Algebra 2, and supplementary materials)

- Define and identify quadratic functions
- Write quadratic functions in standard form ( $f(x) = ax^2 + bx + c$ )
- Identify the shape and graph of a quadratic function
- Simplify square roots
- Factor polynomial expressions
- Solve quadratic equations:
  - by taking the square root of both sides
  - by factoring and applying the zero-product property
  - by completing the square
  - by using the quadratic formula
- Find the axis of symmetry and vertex of a parabola
- Use the discriminant to determine the number of real solutions of a quadratic equation
- Graph parabolas by
  - finding the x-intercepts (zeros or roots)
  - finding the y-intercept
  - finding the vertex
  - finding other points on the graph
- Apply your knowledge to solve application problems involving quadratic functions

#### 5. Complex Numbers (Section 5.6 in Algebra 2)

- add, subtract, multiply and simplify complex numbers
- graph complex numbers on the complex plane
- use the discriminant to determine the nature of the solutions to a quadratic equation
- solve quadratic equations using complex numbers
- evaluate powers of  $i$
- find the magnitude of a complex number

#### 6. Circles (Supplementary materials)

- Define a circle and its associated parts (including radius, chord, diameter, secant, tangent, central angle, inscribed angle, intercepted arc)
- Define and use the degree measure of arcs
- Define and use the length measure of arcs
- Use the “Chords and Arcs” theorem
- Understand the relationship between tangents and radii
- Understand the geometry of a radius perpendicular to a chord
- Use the inscribed angle theorem and related theorems
- Use theorems about measures of intercepted arcs and angles formed by tangents and secants
- Use the equation of a circle with center at the origin

## **7. Exponential and Log Functions (Sections 6.1 – 6.5 in Algebra 2)**

- Evaluate expressions involving fractional and negative exponents
- Solve exponential equations by rewriting the bases to be equal
- Determine the multiplier for exponential growth and decay
- Write and evaluate exponential expressions to model growth and decay situations
- Classify an exponential function as representing exponential growth or exponential decay
- Identify a linear, quadratic, or exponential function
- Use the compound interest formula
- Write equivalent forms for exponential and logarithmic equations
- Graph exponential and logarithmic functions
- Use the definitions of exponential and logarithmic functions to solve equations
- Simplify and evaluate expressions involving logarithms and the properties of logarithms
- Use the common log function to solve exponential and logarithmic equations
- Model and solve real-world problems involving exponential and logarithmic relationships

## **8. Rational Expressions and Equations (packet, also see Sections 8.3, 8.4, 8.5 in Algebra 2)**

- Factor expressions
- Simplify algebraic fractions
- Add, subtract, multiply and divide algebraic fractions
- Solve a rational equation using algebra

## 518 Helpful Formulas for Final Exam

### Arithmetic vs Geometric

**Arithmetic**- to get from one term to the next you are ADDING or SUBTRACTING

Ex. 7, 10, 13, 16, 19 ... ( we are adding 3 each time)

Ex. 25, 21, 17, 13, 9... (we are subtracting 4 each time)

**Geometric** – to get from one term to the next you are MULTIPLYING or DIVIDING

Ex. 4, 12, 36, 108... (we are multiplying by 3 each time)

Ex. 100, 50, 25, 12.5, 6.25... (we are dividing by 2 – or multiplying by  $\frac{1}{2}$  each time)

### Sequence vs Series

**Sequence** = a list a terms

The formula for a sequence gives you ANY TERM you wish to find

**Series** = the SUM of the terms (Add them all up)

The formula for a series gives you the SUM of as many terms as YOU decide

<p><b>Arithmetic Sequence</b></p> <ul style="list-style-type: none"> <li>• Explicit formula <math>t_n = t_1 + (n - 1)d</math></li> <li>• Recursive formula <math>t_n = t_{n-1} + d</math> <math>t_1 = \#</math></li> </ul>	<p><b>Arithmetic Series</b></p> <ul style="list-style-type: none"> <li>• <math>S_n = \frac{n(t_1 + t_n)}{2}</math></li> <li>• <math>\sum_{k=1}^n t_k = t_1 + t_2 + t_3 + \dots + t_n</math></li> </ul>
<p><b>Geometric Sequences</b></p> <ul style="list-style-type: none"> <li>• Explicit formula <math>t_n = t_1 \cdot r^{n-1}</math></li> <li>• Recursive formula <math>t_n = r \cdot t_{n-1}</math> <math>t_1 = \#</math></li> </ul>	<p><b>Geometric Series</b></p> <ul style="list-style-type: none"> <li>• <math>S_n = \frac{t_1(1 - r^n)}{(1 - r)}</math></li> <li>• <math>\sum_{k=1}^n t_k = t_1 + t_2 + t_3 + \dots + t_n</math></li> </ul>

### Growth /Decay Formula

Growth :  $A = P(1 + r)^t$  Decay :  $A = P(1 - r)^t$

Pay attention to whether the problem says growing or decaying...

### Compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

A = Final Amount

P = Principle (= amount you START with)

R = rate  $\rightarrow$  Make sure if it is given to you as a %, change it to a decimal.

Ex. Rate = 4.2%  $\rightarrow$  r = .042 (move the decimal place 2 over to the left twice)

n = number of times COMPOUNDED per year

\*DO NOT USE the COMPOUND formula unless you READ the WORD

“Compounded” in the problem

## Logarithms

### Logs = How to solve for exponents

$$b^x = y \rightarrow x = \log_b y$$

Ex.1 ) Solve for x:  $4^x = 2045$

$$\rightarrow \log_4 2045 = x$$

$$\rightarrow \text{Type into calculator } \frac{\log 2045}{\log 4} = x$$

Ex.2 ) You want to buy a car that cost \$15,000. You invest \$8,000 into a bank account that gives you an APR (annual percentage rate) of 4.5%. How many years will it take you to save up enough money to buy the car?

Set up:

$$A = P(1 + r)^t$$

$$15,000 = 8,000(1 + .045)^t \leftarrow \text{We are trying to solve for the exponent}$$

Divide both sides by 8000

And simplify inside the parenthesis

$$1.875 = (1.045)^t$$

$\leftarrow$  We are now ready to change to log form

$$\rightarrow \log_{1.045} 1.875 = x$$

$$\rightarrow \text{Type into calculator } \frac{\log 1.875}{\log 1.045} = x$$

$x = 14.28 \rightarrow$  so, it will take you **14.28 years** to save up enough money to buy the car (if you don't put any more money in along the way)

### Basic Rules of Exponents

Rule 1: product of two powers with like bases  $b^x \times b^y = b^{(x+y)}$

Rule 2: quotient of two powers with like bases  $\frac{b^x}{b^y} = b^{(x-y)}$

Rule 3: power of a power  $(b^x)^y = b^{(x \cdot y)}$

Rule 4: power with a negative exponent  $b^{-x} = \frac{1}{b^x}$

Rule 5: power with a zero exponent  $b^0 = 1$

Rule 6: power of a quotient  $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

## Quadratic Formula

When you are given a quadratic trinomial:  $ax^2 + bx + c = 0$

You can solve for x with the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- this will only work if your equation = 0
- This will only work if you line up your equation in the order  $ax^2 + bx + c = 0$

The imaginary #

$$i = \sqrt{-1} = i$$

$$i^2 = (\sqrt{-1})^2 = -1$$

$$i^3 = (i)(\sqrt{-1})^2 = (i)(-1) = -i$$

$$i^4 = (\sqrt{-1})^2(\sqrt{-1})^2 = (-1)(-1) = 1$$

$$i^5 = i$$

$$i^6 = -1$$

$$i^7 = -i \quad \text{This pattern continues every}$$

group of 4

$$i^8 = 1$$

Ex. What is the  $\sqrt{-49}$ ?

$$\sqrt{-49} = \sqrt{49} \cdot \sqrt{-1} = 7i$$

## Complete the Square

$X^2 - 6x + 11 = 0$  ← what do you want “c” value to be? 9!

So -2 from both sides

$$X^2 - 6x + 11 = 0$$

$$\quad -2 \quad -2$$

$$X^2 - 6x + 9 = -2$$

$(x - 3)^2 = -2$  ← square root both sides

$$x - 3 = \pm\sqrt{-2}$$

$$x = 3 \pm \sqrt{-2}$$

$$x = 3 \pm i\sqrt{2}$$

# 518 Final Exam Review Packet

## SEQUENCES AND SERIES (Sections 11.1 – 11.5 in Algebra 2)

- Determine whether the following sequences are arithmetic, geometric or neither.  
If so identify the common difference or common ratio.
  - 54, 18, 6, 2, ... \_\_\_\_\_
  - 62, 66, 70, 74, ... \_\_\_\_\_
  - 2, -5, 25, ... \_\_\_\_\_
- Find  $t_{15}$  of the sequence given by  $t_1 = -4$  and  $d = 3$ .
- For the sequence 3, 6, 12, 24, ...
  - Find  $t_{20}$
  - Find  $S_{12}$
  - Write a recursive formula for this sequence.
- Find  $S_{20}$  for the series  $9 + 2 - 5 - 12 - 19 - \dots$
- Find  $33 + 38 + 43 + \dots + 123$
- Find three arithmetic means between 1 and  $-27$



7. Find four geometric means between 4 and 972.
8. Boxes are stacked with 1 in the top row, 5 in the second row, 9 in the third row, etc.
- How many boxes are in the eighteenth row?
  - What is the total number of boxes?
9. A car that costs \$ 26,500 depreciates and its value at the end of a given year is 90% of its value at the end of the preceding year. Find the value of the car after 8 years.
10. Evaluate:
- $\sum_{k=1}^4 (2k - 3)$
  - $\sum_{k=1}^{10} 4(5^{k-1})$
  - $\sum_{x=1}^{99} (3x - 2)$
11. a) What is the 101<sup>st</sup> term of 7, 13, 19, 25, ...?
- b) What is the sum of the first 101 terms?

## FUNCTIONS

(Sections 2.3 – 2.5 in Algebra 2)

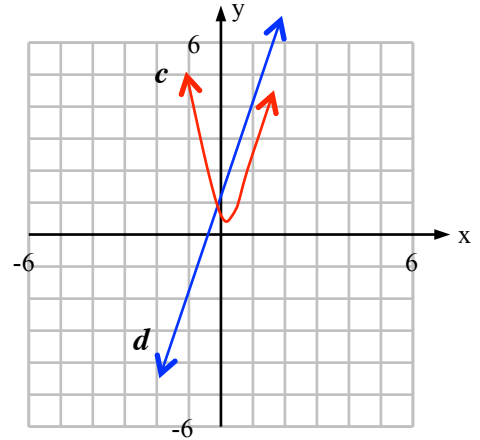
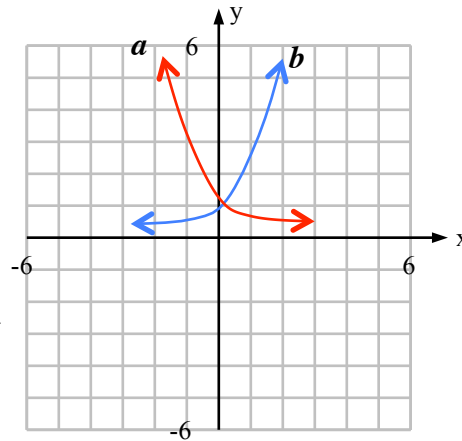
1. Match the equation with its graph. Then state whether the function is linear, quadratic, or exponential.

$y = 3x + 1$  \_\_\_\_\_

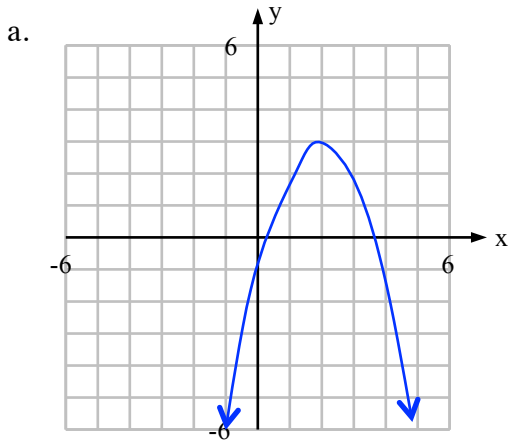
$y = 3^x$  \_\_\_\_\_

$y = 3x^2 - 2x + 1$  \_\_\_\_\_

$y = \left(\frac{1}{3}\right)^x$  \_\_\_\_\_



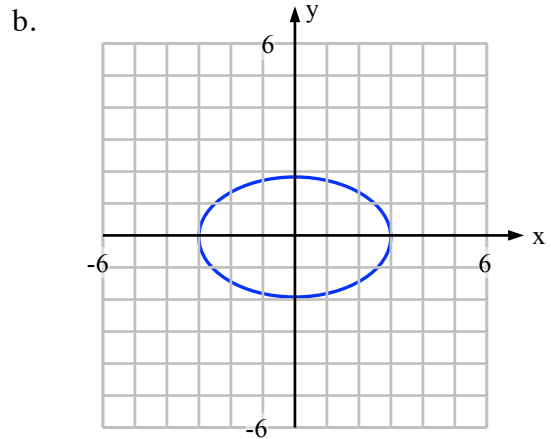
2. Find the domain and range. Is it a function?



Domain:

Range:

Function:



Domain:

Range:

Function:

3. a. Let  $h(x) = x^4 - x^3 + x$ .

Find  $h(-2)$ . \_\_\_\_\_

Find  $h(b)$ . \_\_\_\_\_

b. Let  $q(x) = \log_2 x$ .

$$q(32) = \underline{\hspace{2cm}} \quad q(1/2) = \underline{\hspace{2cm}} \quad q(\sqrt[3]{2}) = \underline{\hspace{2cm}}$$

4. Given:  $f(x) = 2x - 1$  and  $g(x) = 4x + 3$

a.  $f(g(3))$

b.  $g(g(3))$

c.  $f(g(x))$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

d.  $(f - g)(x)$

e.  $f \cdot g(x)$

f.  $\left(\frac{f}{g}\right)(x)$

[Don't forget domain restriction.]

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_ ; \_\_\_\_\_

5. Write the inverse function of:

a.  $f(x) = 15 - x$

b.  $y = 3x + 5$

\_\_\_\_\_

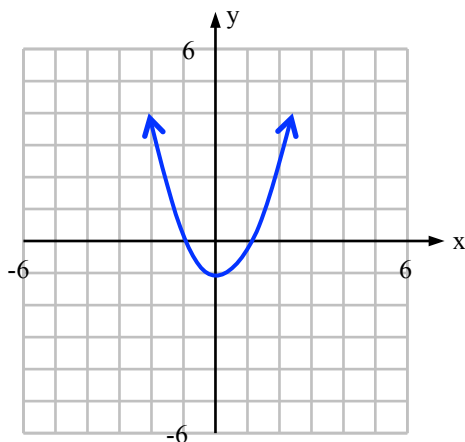
\_\_\_\_\_

6. Given  $g(x) = x^2 - 1$

Use the equation or graph to find;

a.  $g(-2) =$

b. all  $x$ -values for which  $g(x) = 0$



# QUADRATIC FUNCTIONS and COMPLEX NUMBERS

(Sections 5.1 – 5.6 in Algebra 2)

1. Solve. Domain is the set of complex numbers. Find all solutions. Leave in simplified radical form if answer is irrational.

a.  $x^2 - 6x + 11 = 0$   
(Complete the Square)

\_\_\_\_\_

b.  $4x^2 - 7x = 15$   
(Quadratic Formula)

\_\_\_\_\_

c.  $x^2 - 11x - 12 = 0$   
(Factor)

\_\_\_\_\_

d.  $2(x - 3)^2 = 32$   
(Square Root)

\_\_\_\_\_

Choose a method and solve.

e.  $x^2 - 25 = 0$

\_\_\_\_\_

f.  $x^2 + 25 = 0$

\_\_\_\_\_

2. Does  $y = -2x^2 + 3x$  have a maximum or minimum point? \_\_\_\_\_

3. Expand and identify “a,” “b,” and “c” for:  $y = (2x - 3)(x + 2)$

Expand: \_\_\_\_\_ a = \_\_\_\_\_ b = \_\_\_\_\_ c = \_\_\_\_\_

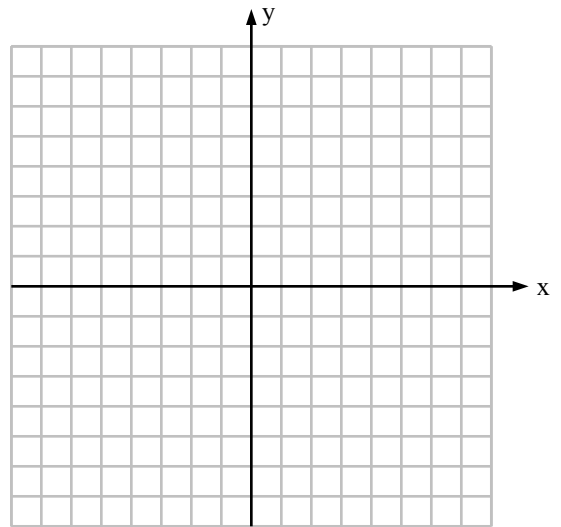
4. Graph:  $y = x^2 - 7x + 10$

Axis of symmetry: \_\_\_\_\_

Vertex: \_\_\_\_\_

y-intercept: \_\_\_\_\_

x-intercepts: \_\_\_\_\_



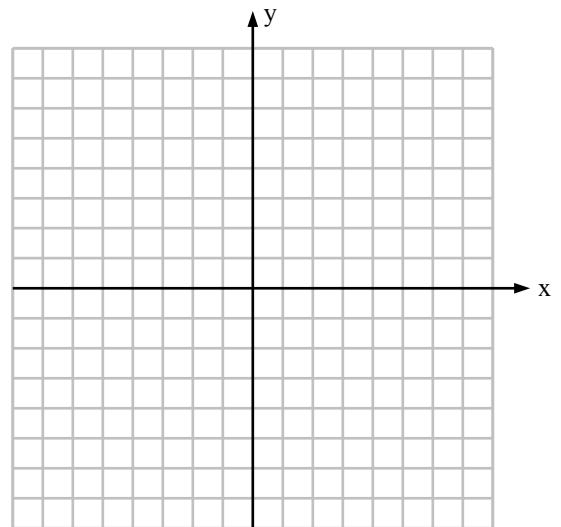
5. Graph:  $y = -x^2 + 5x$

Axis of symmetry: \_\_\_\_\_

Vertex: \_\_\_\_\_

y-intercept: \_\_\_\_\_

x-intercepts: \_\_\_\_\_



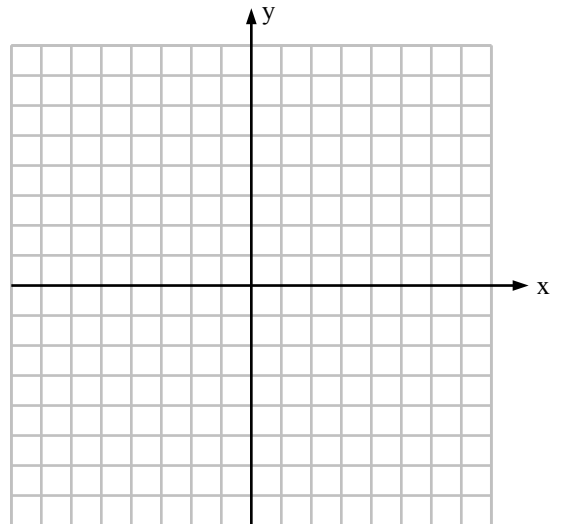
6. Graph:  $y = 2x^2 - 12x + 19$

Axis of symmetry: \_\_\_\_\_

Vertex: \_\_\_\_\_

y-intercept: \_\_\_\_\_

x-intercepts: \_\_\_\_\_



7. The height of a baseball thrown up in the air with an initial velocity of 20 meters per second is given by:  $h(t) = 20t - 5t^2$

a. At what time(s) does the baseball reach a height of 6 meters (to the nearest tenth of a second)?

b. When does the baseball hit the ground?

c. What is the maximum height of the baseball?

d. When does the baseball reach this maximum height?

e. How high is the baseball after 1.5 seconds?

8. Simplify:

a.  $(5 - 2i)^2$

\_\_\_\_\_

b.  $\sqrt{-49}$

\_\_\_\_\_

c.  $\sqrt{-4}\sqrt{-9}$

\_\_\_\_\_

d.  $i^{47}$

\_\_\_\_\_

e.  $6\sqrt{-75}$

\_\_\_\_\_

f.  $3i^3 + 4i^4$

\_\_\_\_\_

g.  $(2 - 6i) - (5 + 14i)$

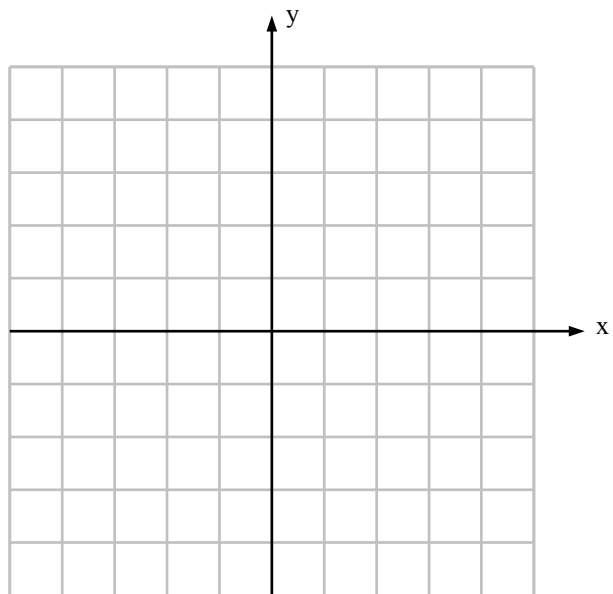
\_\_\_\_\_

h.  $(4 + 3i) + (4 - 3i)$

\_\_\_\_\_

9. a. Graph:  $3 - 4i$ .

b. Find the distance from  $3 - 4i$  to the origin (the magnitude of  $3 - 4i$ ).

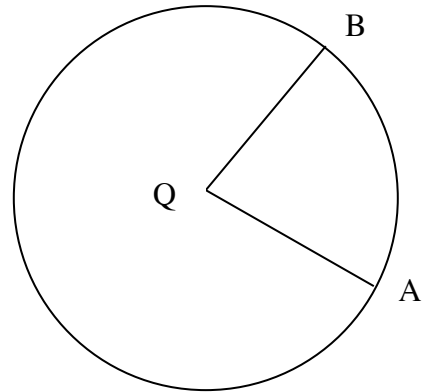


# CIRCLES

1. In the circle Q,  $m\angle AQB = 72^\circ$  and the diameter is 8 inches.

a. What is the length of arc AB?

b. What is the  $m\widehat{AB}$ ?

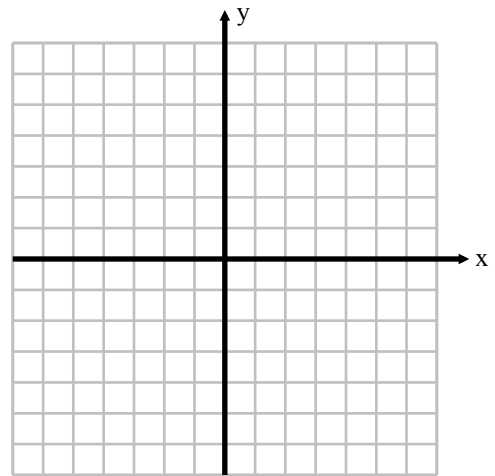


3. a. State the center and radius of  $x^2 + y^2 = 12$ .

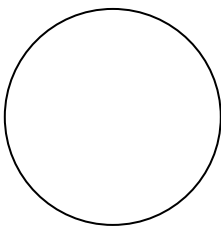
Center = \_\_\_\_\_ Radius = \_\_\_\_\_

b. State the intercepts of  $x^2 + y^2 = 12$ .

c. Graph  $x^2 + y^2 = 12$ .

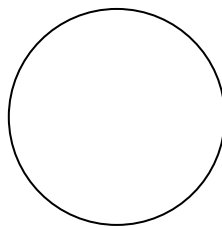


3.a.



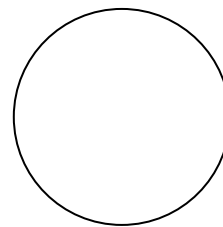
$m\angle BAE = 75^\circ$   
 $m\angle BDE = \underline{\hspace{2cm}}$

b.



Q is the center  
 $m\angle FBD = 70^\circ$   
 $m\widehat{BF} = \underline{\hspace{2cm}}$

c.



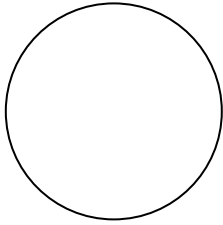
Q is the center  
 $m\angle BQC = 86^\circ$   
 $m\angle BAC = \underline{\hspace{2cm}}$

4. What is a major arc? What is a minor arc?

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5.  
a.

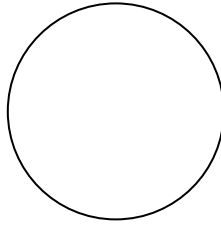


$$m\angle HJ = 80^\circ$$

$$m\angle HNJ = 62^\circ$$

$$m\angle KL = \underline{\hspace{2cm}}$$

b.

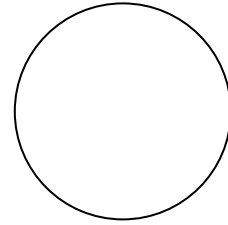


$$m\angle BAE = 75^\circ$$

$$m\angle CD = 25^\circ$$

$$m\angle BE = \underline{\hspace{2cm}}$$

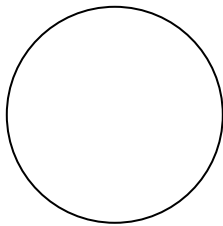
c.



$$m\angle FBD = 70^\circ$$

$$m\angle BD = \underline{\hspace{2cm}}$$

d.

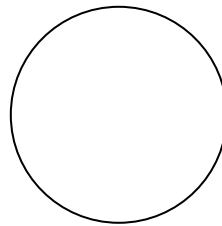


$$\text{arc } AB = 120^\circ$$

$$m\angle BCD = 140^\circ$$

$$\text{arc } AD = \underline{\hspace{2cm}}$$

e.

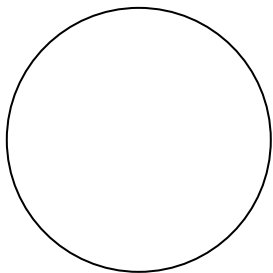


Q is the center of the circle

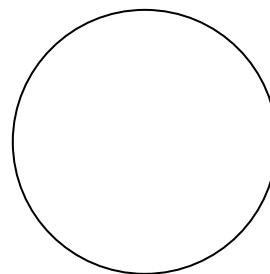
$$m\angle AQB = 100^\circ$$

$$m\angle QBA = \underline{\hspace{2cm}}$$

6. The diameter in circle Q is 10.  
Chord CD = 8. Find the distance  
between the two chords.



7. LM is tangent to circle Q.  
NL = 3, ML = 10. Find the radius.



8. Given:

Circle Q

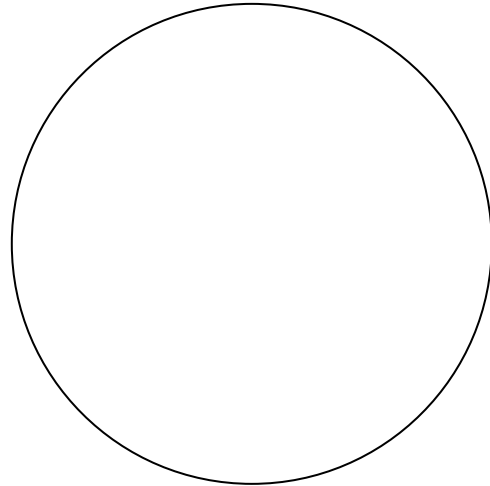
Diameter DB

CF tangent at C

GA tangent at A

$$m\widehat{AB} = 30^\circ$$

$$m\widehat{DC} = 80^\circ$$



a. Find the following measures:

$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 6 = \underline{\hspace{2cm}}$$

$$m\widehat{BD} = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

$$m\angle 7 = \underline{\hspace{2cm}}$$

$$m\widehat{CB} = \underline{\hspace{2cm}}$$

$$m\angle 3 = \underline{\hspace{2cm}}$$

$$m\angle 8 = \underline{\hspace{2cm}}$$

$$m\widehat{BDC} = \underline{\hspace{2cm}}$$

$$m\angle 4 = \underline{\hspace{2cm}}$$

$$m\angle 9 = \underline{\hspace{2cm}}$$

$$m\angle 5 = \underline{\hspace{2cm}}$$

$$m\angle 10 = \underline{\hspace{2cm}}$$

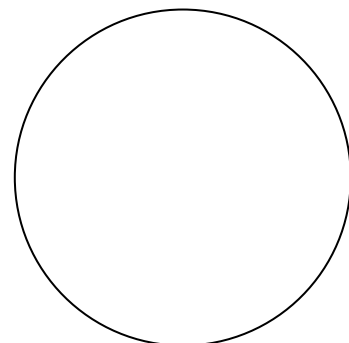
b. Using the diagram in #6, name a

radius \_\_\_\_\_ chord \_\_\_\_\_ central angle \_\_\_\_\_

major arc \_\_\_\_\_ minor arc \_\_\_\_\_ tangent \_\_\_\_\_

inscribed angle \_\_\_\_\_

9. The circle with center O has a diameter of 20.  $\overline{OD}$  is 8. Find the length of the chord  $\overline{AB}$ .



## EXPONENTS AND LOGARITHMS

(Sections 2.2, 6.1 - 6.5 in Algebra 2)

1. Simplify. Write your answers with positive exponents.

a.  $(3x^3)^4(10xy^8)$

b.  $x^5x^{-3}$

c.  $(-2x^3y^{-2})^3$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

d.  $\left(\frac{6x^7}{2x^3}\right)^{-2}$

e.  $\frac{16x^6yz^2}{-4xy^5z^2}$

f.  $\frac{m^{-1}n^2}{n^{-3}}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

2. Solve without a calculator.

a.  $2^x = 64$

b.  $3^x = \frac{1}{81}$

c.  $8^x = 128$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

3. Simplify without a calculator. Leave answers exact.

a.  $9^{\frac{1}{2}}$

b.  $4^0$

c.  $6^{-2}$

d.  $-3^2$

e.  $(-3)^2$

f.  $\left(\frac{16}{25}\right)^{\frac{1}{2}}$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

g.  $8^{\frac{2}{3}}$

h.  $9^{-\frac{1}{2}}$

i.  $\left(\frac{4}{9}\right)^{-2}$

j.  $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

k.  $(3^{\sqrt{2}})^{\sqrt{2}}$

l.  $\left(\frac{4}{5}\right)^{-2}$

\_\_\_\_\_

\_\_\_\_\_

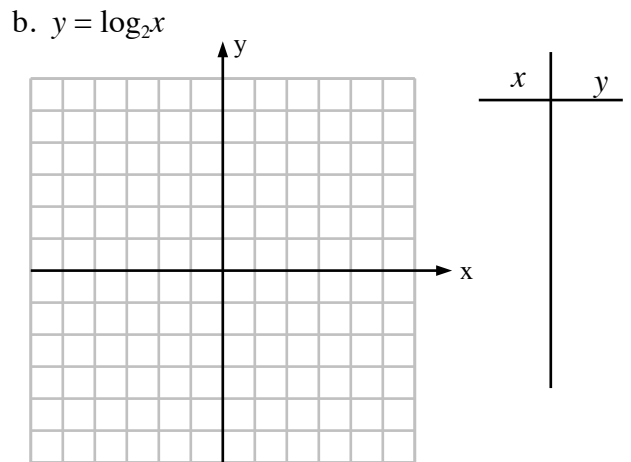
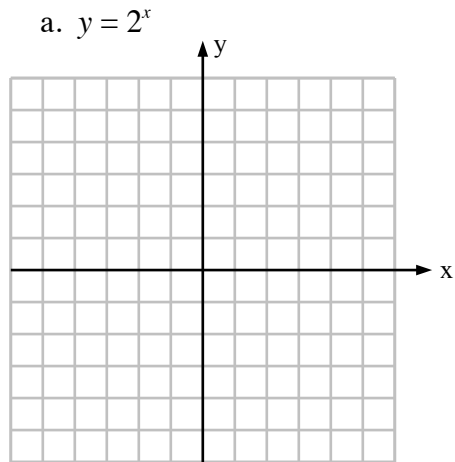
\_\_\_\_\_

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\_\_\_\_\_

\_\_\_\_\_

4. Graph accurately. Make a table of values if necessary. Label your graphs and your scale.



5. Write in logarithmic form.

$2^5 = 32$  \_\_\_\_\_

$10^{-2} = .01$  \_\_\_\_\_

6. Write in exponential form.

$\log_3 81 = 4$  \_\_\_\_\_

$\log 100 = 2$  \_\_\_\_\_

7. Write in exponential form:

a.  $\sqrt{2}$

b.  $\sqrt{5^3}$

c.  $\sqrt[6]{a^3}$

d.  $\sqrt[3]{x}$

e.  $\sqrt[4]{y^3}$

8. Write in radical form:

a.  $9^{\frac{1}{3}}$

b.  $5x^{\frac{1}{2}}$

c.  $b^{\frac{3}{4}}$

9. Solve. If necessary, round answers to two decimal places.

a.  $3^{2x} = 27$

b.  $x^{\frac{2}{3}} = 16$

c.  $2^x = 12$

d.  $\log_x 64 = 6$

e.  $\log_5 x = 2.5$

f.  $\log 4x = 2$

g.  $\log_3 200 = x$

h.  $6^{x+2} = 150$

i.  $11 + 5^x = 360$

10. Write as a single log:

a.  $\log 4a + \log 9$  \_\_\_\_\_

b.  $\log_9 30 - 6\log_9 y$  \_\_\_\_\_

11. True or false:

a.  $\log_2 1 = \log_3 1$

b.  $\log(2 \cdot 3) = \log 2 \cdot \log 3$

c.  $\log A^x = x \log A$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

12. Predict the population of bacteria given 70 bacteria that double every 30 minutes

a. after 1 hour....

b. after 6 hours...

13. The population of Tokyo, Japan was about 28,447,000 in 1995 and was projected to grow at an annual rate of 1.1%.

- a. Predict the population, to the nearest person, for the year 2010.
  
  
  
  
  
  
  
  
  
  
- b. Assuming the growth rate stays the same, when will the population be 40,000,000?

14. Find the final amount of a \$2000 investment after 5 years at 8% annual interest

- a. compounded quarterly...
  
  
  
  
  
  
  
- b. compounded daily ...
  
  
  
  
  
  
  
- c. compounded annually...

15. The value of a painting was \$12,000 in 2005 and increases by 9% of its value each year.

- a. Write and evaluate an expression to estimate the painting's value in 2020.
  
  
  
  
  
  
  
  
  
  
- b. In how many years will it be worth \$60,000? Round to the nearest hundredth.

16. Tell whether each function represents exponential growth or decay.

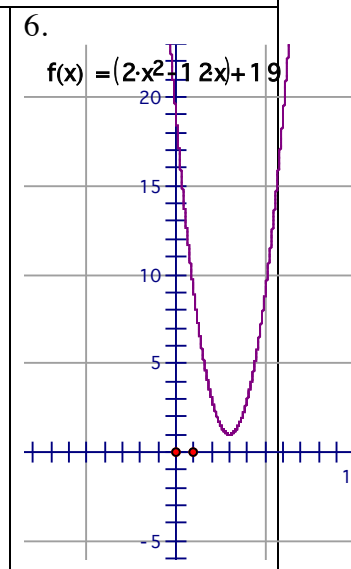
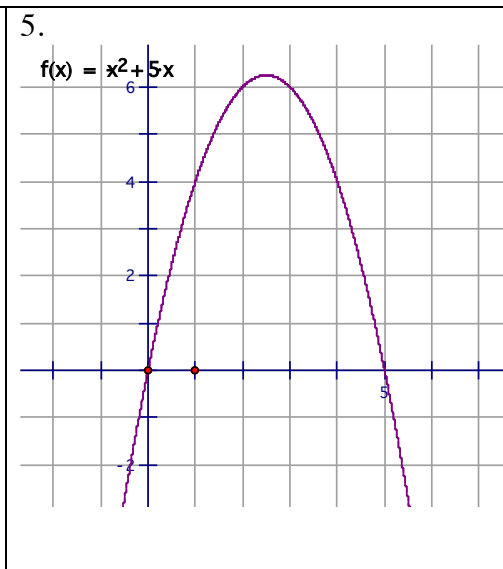
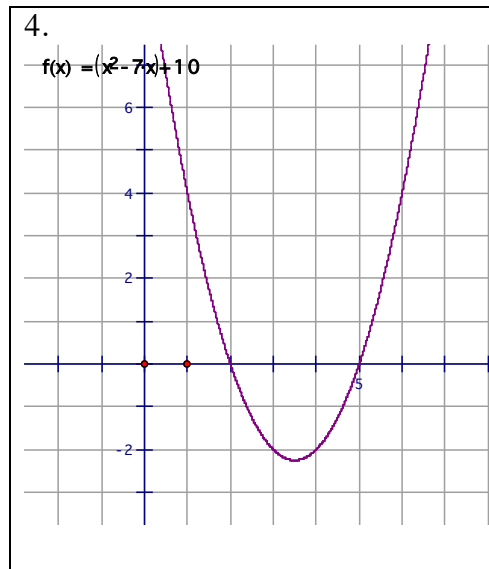
- a.  $y = 8(1.5)^x$
- b.  $y = 500(.5)^x$
- c.  $y = .25(5)^x$

## 518 Final Exam Review Solutions

<i>Sequences and Series</i>	<i>Functions</i>
1. a) geometric, $r = 1/3$ b) arithmetic, $d = 4$ c) neither	1. $y = 3x + 1$ is linear, graph (d). $y = 3^x$ is exponential, graph (b). $y = 3x^2 - 2x + 1$ , graph (c). $y = \frac{1}{3}^x$ is exponential, graph (a).
2. 38	2. a) Domain = all reals    Range is $y \leq 3$ .  Yes it is a function.  You do not have one-to-one mapping because the inverse is not a function.
3. a) 1,572,864  b) 12,285  c) $t_n = 2t_{n-1}$ , $t_1 = 3$	b) Domain is $-3 \leq x \leq 3$ . Range is $-2 \leq y \leq 2$ . It is not a function.
4. $S_{20} = -1150$	3. a) $h(-2) = 22$ $h(b) = b^4 - b^3 - b$
5. $S_{19} = 1482$	b) $q(32) = 5$ $q(1/2) = -1$ $q(\sqrt[3]{2}) = \frac{1}{3}$
6. <u>-6</u> , <u>-13</u> , <u>-20</u>	4. a) 29    b) 63    c) $8x + 5$ d) $-2x - 4$
7. <u>12</u> , <u>36</u> , <u>108</u> , <u>324</u>	e) $8x^2 + 2x - 3$ f) $\frac{2x-1}{4x+3}$ and $x \neq -.75$
8. a) $t_{18} = 69$  b) $S_{18} = 630$	5. a) $y = -1x + 15$ or $y = 15 - x$ b) $y = \frac{1}{3}x - \frac{5}{3} = \frac{x-5}{3}$
9. \$11,407.38	6. a) 3    b) $x = 1$ or $-1$
10. a) 8  b) 9,765,624  c) 14,652	
11. a) $t_{101} = 607$  b) $S_{101} = 31,007$	

## Quadratics

- $x = 3 \pm i\sqrt{2}$
  - $x = 3$  or  $-1.25$
  - $x = 12$  or  $-1$
  - $x = 7$  or  $-1$
  - $x = 5$  or  $-5$
  - $x = 5i$  or  $-5i$
- maximum
- $2x^2 + 1x - 6$  and  $a = 2$ ,  $b = 1$ ,  $c = -6$
- Axis of sym. is  $x = 3.5$ . The vertex is  $(3.5, -2.25)$ . The y-intercept is  $(0, 10)$  and the x-intercepts are  $(2, 0)$  and  $(5, 0)$ . See graph below.
- The axis of sym. is  $x = 2.5$ . The vertex is  $(2.5, 6.25)$ . The y-intercept is  $(0, 0)$  and the x-intercepts are  $(5, 0)$  and  $(0, 0)$ . See graph below.
- The axis of sym. is  $x = 3$ . The vertex is  $(3, 1)$ . The y-intercept is  $(0, 19)$  and there are no x-intercepts. See graph below.
- .33 sec or 3.67 sec.
  - 4 sec.
  - 20 m
  - 2 sec
  - 18.75 m
- $21 - 20i$
  - $7i$
  - $-6$
  - $-i$
  - $30i\sqrt{3}$
  - $4 - 3i$
  - $-3 - 20i$
  - 25
- Plot the point  $(3, -4)$ .
  - 5





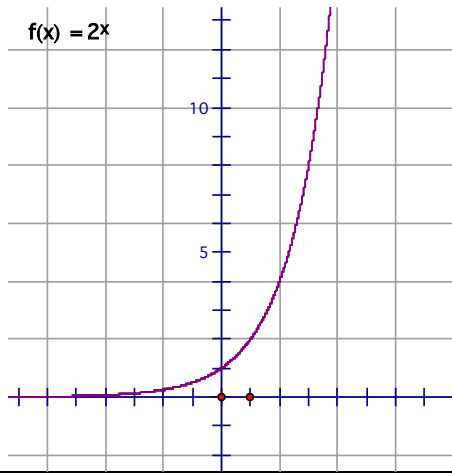
**Circles**

1. a) 5.03 in      b)  $72^\circ$
2. Center (0,0)  
Radius = 3.5 or  $\sqrt{12}$  or  $2\sqrt{3}$   
Intercepts are (0, 3.5); (0,-3.5); (3.5,0)  
(-3.5,0)
3. a)  $105^\circ$       b)  $40^\circ$       c)  $43^\circ$
4. A major arc is more than  $180^\circ$ . A minor arc is less than  $180^\circ$ .
5. a)  $44^\circ$       b)  $175^\circ$       c)  $140^\circ$       d)  $160^\circ$   
e)  $40^\circ$
6. 3
7. 15.17
8.  $m\angle 1 = 80^\circ$   
 $m\angle 2 = 100^\circ$   
 $m\angle 3 = 65^\circ$   
 $m\angle 4 = 25^\circ$   
 $m\angle 5 = 50^\circ$   
 $m\angle 6 = 40^\circ$   
 $m\angle 7 = 115^\circ$   
 $m\angle 8 = 65^\circ$   
 $m\angle 9 = 50^\circ$   
 $m\angle 10 = 55^\circ$   
 $m \text{ arc } BD = 180^\circ$   
 $m \text{ arc } CB = 100^\circ$   
 $m \text{ arc } BDC = 260^\circ$
9. Chord  $\overline{AB}$  is 12 units long.

**Exponents and logs**

1. a)  $810x^{13}y^8$       b)  $x^2$       c)  $-8x^9/y^6$   
d)  $\frac{1}{9x^8}$       e)  $\frac{-4x}{y^4}$       f)  $\frac{n^5}{m}$
2. a) 6      b) -4      c)  $7/3$
3. a) 3      b) 1      c)  $\frac{1}{36}$       d) -9      e) 9      f)  $\frac{4}{5}$   
g) 4      h)  $\frac{1}{3}$       i)  $\frac{81}{16}$       j)  $\frac{2}{3}$       k) 9      l)  $\frac{25}{16}$
4. See graphs on next page.
5. a)  $\log_2 32 = 5$       b)  $\log .01 = -2$
6. a)  $3^4 = 81$       b)  $10^2 = 100$
7. a)  $2^{\frac{1}{2}}$       b)  $5^{\frac{3}{2}}$       c)  $a^{\frac{1}{2}}$  or  $a^5$       d)  $x^{\frac{1}{3}}$       e)  $y^{\frac{3}{4}}$
8. a)  $\sqrt[3]{9}$       b)  $5\sqrt{x}$       c)  $\sqrt[4]{b^3}$  or  $(\sqrt[4]{b})^3$
9. a)  $x = 1.5$       b)  $x = 64$       c)  $x = 3.58$   
d)  $x = 2$       e)  $x = 55.90$       f)  $x = 25$   
g)  $x = 4.82$       h)  $x = .80$       i)  $x = 3.64$
10. a)  $\log 36a$       b)  $\log_9 \frac{30}{y^6}$
11. a) true      b) false      c) true
12. a) 280 bacteria      b) 286,720 bacteria
13. a) 33,519,984      b) 31.1 years from 1995
14. a) \$2971.89      b) \$2983.52      c) \$2938.66
15. a) \$43,709.79      b) 18.68 years
16. a) growth      b) decay      c) growth

4a.  $y = 2^x$



4b.  $y = \log_2 x$

