AGENDA

Objective:SWBAT simplify and evaluate expressions involving logarithms and **properties** of logarithms

Language Objective: SWBAT identify the error in an a sample problem by writing an explanation of the error as well as the correct method for solving the problem in a Think...Write..Pair...Share activity with a partner

1) Take out HW to be checked
2) Do Now
3) Discover: Properties of Logarithms
   Groups: product & quotient properties
   Class: Power property
4)Try some! (practice)
5) Think...Write...Pair...Share
6) Practice
7) Exit Ticket

HW: "Logs worksheet #2"

- Write **equivalent forms** for **exponential** and **logarithmic** equations
- Use the **common log** function to solve exponential and logarithmic equations
- **Simplify and evaluate expressions** involving logarithms and the **properties** of logarithms
- Use the definitions of exponential and logarithmic functions **to solve equations**
- **Graph** exponential and logarithmic functions
- **Model and solve real-world problems** involving exponential and logarithmic relationships

DO NOW!

1) Write in logarithmic form: $4^2 = 16 \rightarrow$ _____

2) Write in exponential form: $\log_7(1/49) = -2 \rightarrow$ _____

3) Solve for the variable: (*Hint* - switch to **exponential** form.)

a) $\log_2 16 = x$ b) $\log_9 n = 2$ c) $\log_b 1{,}000{,}000 = 6$

d) $\log_4 n = 0$ e) $\log_7 1 = x$ f) $\log_b(1/8) = 3$

- Write **equivalent forms** for **exponential** and **logarithmic** equations
- Use the **common log** function to solve exponential and logarithmic equations
- **Simplify and evaluate expressions** involving logarithms and the **properties** of logarithms
- Use the definitions of exponential and logarithmic functions **to solve equations**
- **Graph** exponential and logarithmic functions
- **Model and solve real-world problems** involving exponential and logarithmic relationships

DO NOW!

1) Write in logarithmic form: $4^2 = 16 \rightarrow$ $\log_4 16 = 2$

2) Write in exponential form: $\log_7 (1/49) = -2 \rightarrow$ $7^{-2} = 1/49$

3) Solve for the variable: (*Hint* - switch to **exponential** form.)

a) $\log_2 16 = x$

$2^x = 16$

$x = 4$

b) $\log_9 n = 2$

$9^2 = n$

$n = 18$

c) $\log_b 1{,}000{,}000 = 6$

$b^6 = 1{,}000{,}000$

$b = 10$

d) $\log_4 n = 0$

$4^0 = n$

$n = 1$

e) $\log_7 1 = x$

$7^x = 1$

$x = 0$

f) $\log_b (1/8) = 3$

$b^3 = 1/8$

$b = 2$

Math 518 Name _____ Date _____

Exploring Properties of Logarithms

1. Complete the table:

x	2	4	8	16	32	64	128
$\log_2 x$							

2. The expression $\log_2(2\cdot4)$ can be written as $\log_2 8$. Use this fact and your table above to complete the following:

a. $\log_2(2\cdot4) = \log_2 8 =$ ____ $\log_2 2 + \log_2 4 =$ ____ + ____ = ____

b. $\log_2(2\cdot8) = \log_2$ ____ = ____ $\log_2 2 + \log_2 8 =$ ____ + ____ = ____

c. $\log_2(2\cdot16) = \log_2$ ____ = ____ $\log_2 2 + \log_2 16 =$ ____ + ____ = ____

d. $\log_2(2\cdot32) = \log_2$ ____ = ____ $\log_2 2 + \log_2 32 =$ ____ + ____ = ____

3. How is the first expression in each pair related to the second?

Use this pattern to make a conjecture about:

$\log_2(a\cdot b) =$ _____

Math 518 Name _____ Date _____

Exploring Properties of Logarithms

1. Complete the table:

x	2	4	8	16	32	64	128
$\log_2 x$	1	2	3	4	5	6	7

2. The expression $\log_2(2\cdot4)$ can be written as $\log_2 8$. Use this fact and your table above to complete the following:

a. $\log_2(2\cdot4) = \log_2 8 =$ 3 $\qquad \log_2 2 + \log_2 4 =$ 1 + 2 = 3

b. $\log_2(2\cdot8) = \log_2$ 16 $=$ 4 $\qquad \log_2 2 + \log_2 8 =$ 1 + 3 = 4

c. $\log_2(2\cdot16) = \log_2$ 32 $=$ 5 $\qquad \log_2 2 + \log_2 16 =$ 1 + 4 = 5

d. $\log_2(2\cdot32) = \log_2$ 64 $=$ 6 $\qquad \log_2 2 + \log_2 32 =$ 1 + 5 = 6

3. How is the first expression in each pair related to the second?

They are equal!

Use this pattern to make a conjecture about:

$\log_2(a\cdot b) =$ $\log_2 a + \log_2 b$

4. The expression $\log_2 \frac{16}{2}$ can be written as $\log_2 8$. Use this fact and your table above to complete the following:

a. $\log_2 \frac{16}{2} = \log_2 8 =$ ____ $\log_2 16 - \log_2 2 =$ ____ - ____ = ____

b. $\log_2 \frac{64}{32} = \log_2$ ____ = ____ $\log_2 64 - \log_2 32 =$ ____ - ____ = ____

c. $\log_2 \frac{32}{8} = \log_2$ ____ = ____ $\log_2 32 - \log_2 8 =$ ____ - ____ = ____

d. $\log_2 \frac{8}{4} = \log_2$ ____ = ____ $\log_2 8 - \log_2 4 =$ ____ - ____ = ____

5. How is the first expression in each pair related to the second?

Use this pattern to make a conjecture about:

$\log_2\left(\frac{a}{b}\right) =$ _____

6. Using your two new conjectures, write each expression as a single logarithm. Then simplify if possible.

a. $\log_3 10 - \log_3 5$ b. $\log_4 18 + \log_4 2$

c. $\log_5 3 + \log_5 6 + \log_5 9$ d. $\log_5 15 - \log_5 3$

e. $\log_b u + \log_b v - \log_b uw$ f. $\log_b 4x - \log_b 3y + \log_b y$

4. The expression $\log_2 \frac{16}{2}$ can be written as $\log_2 8$. Use this fact and your table above to complete the following:

a. $\log_2 \frac{16}{2} = \log_2 8 =$ 3 $\qquad \log_2 16 - \log_2 2 =$ 4 - 1 = 3

b. $\log_2 \frac{64}{32} = \log_2$ 2 = 1 $\qquad \log_2 64 - \log_2 32 =$ 6 - 5 = 1

c. $\log_2 \frac{32}{8} = \log_2$ 4 = 2 $\qquad \log_2 32 - \log_2 8 =$ 5 - 3 = 2

d. $\log_2 \frac{8}{4} = \log_2$ 2 = 1 $\qquad \log_2 8 - \log_2 4 =$ 3 - 2 = 1

5. How is the first expression in each pair related to the second?

Use this pattern to make a conjecture about:

$\log_2\left(\frac{a}{b}\right) =$ $\log_2 a - \log_2 b$

6. Using your two new conjectures, write each expression as a single logarithm. Then simplify if possible.

a. $\log_3 10 - \log_3 5$

$\log_3 2$

b. $\log_4 18 + \log_4 2$

$\log_4 9$

c. $\log_5 3 + \log_5 6 + \log_5 9$

$\log_5 162$

d. $\log_5 15 - \log_5 3$

$\log_5 5 = 1$

e. $\log_b u + \log_b v - \log_b uw$

$\log_b(uv/uw)$

$= \log_b(v/w)$

f. $\log_b 4x - \log_b 3y + \log_b y$

$\log_b(4x \cdot y/3y)$

$\log_b(4x/3)$

Properties of Logs

Product Property	$\log_b(m \cdot n) = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b(m^p) = p \cdot \log_b m$

where $m > 0, n > 0$, b > 0 and b ≠1
(all values are positive and b ≠1)

Why do you think *this* is?
Lets *prove* the rule together!

$$b^x = m$$

Properties of Logs

Product Property	$\log_b(m \cdot n) = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b(m^p) = p \cdot \log_b m$

where $m > 0, n > 0$, b > 0 and b ≠1
(all values are positive and b ≠1)

Why do you think *this* is?
Lets *prove* the rule together!

b^x

$$b^x = m \rightarrow log_b m = x$$

$$(b^x)^p = (m)^p$$

$$b^{x \cdot p} = m^p$$

$$log_b(m^p) = x \cdot p = p \cdot x$$

$$log_b(m^p) = x \cdot p = p \cdot log_b m$$

Try Some!

Problem 1) Rewrite log12 + log5 as a single term using the **product rule formula**

Answer

Problem 2) Rewrite $\log_3 12 + \log_3 11$ using the **product rule formula**

Answer

Problem 3) Rewrite $\log_5 11 + \log_5 a$ using the **product rule formula**

Answer

Practice problem 1) Rewrite log20 – log5 as a single term using the **quotient rule formula**

Answer

Practice problem 2) Rewrite $\log_2 100 - \log_2 25$ as a single term using the **quotient rule formula**

Answer

Practice problem 3) Rewrite $\log_2 40 - \log_2 5$ as a single term using the **quotient rule formula**

Answer

Practice problem 1) Rewrite $\log_3 x^2$ as a single term using the **power rule formula**

Answer

Practice problem 2) Rewrite $\log_3 9^x$ as a single term using the **power rule formula**

Answer

Try Some! (Answers)

Problem 1) Rewrite log12 + log5 as a single term using the **product rule formula**
log12 + log5 = log(12*5) = log60

Problem 2) Rewrite $\log_3 12 + \log_3 11$ using the **product rule formula**
$\log_3 12 + \log_3 11 = \log_3(12 * 11) = \log_3 132$

Problem 3) Rewrite $\log_5 11 + \log_5 a$ using the **product rule formula**
$\log_5 11 + \log_5 a = \log_5(11*a) = \log_5 11a$

Practice problem 1) Rewrite log20 – log5 as a single term using the **quotient rule formula**

$$log20 - log5 = log(\frac{20}{5}) = log4$$

Practice problem 2) Rewrite $\log_2 100 - \log_2 25$ as a single term using the **quotient rule formula**

$$log_2(100) - log_2(25) = log_2(\frac{100}{25}) = log_2(4).$$

$\log_2 4$ is a **logarithm equation that you can solve** and get an answer of 2

Practice problem 3) Rewrite $\log_2 40 - \log_2 5$ as a single term using the **quotient rule formula**

$$log_2(40) - log_2(5) = log_2(\frac{40}{5}) = log_2(8).$$

$\log_2 8$ is a **logarithm equation that you can solve** and get an answer of 3

Practice problem 1) Rewrite $\log_3 x^2$ as a single term using the **power rule formula**
$\log_3 x^2 = 2\log_3 X$.

Practice problem 2) Rewrite $\log_3 9^x$ as a single term using the **power rule formula**
$\log_3 9^x = x\log_3 9$.

$\log_3 9$ can be **solved as a logarithmic equation** . $\log_3 9 = 2$

Therefore, the final answer is x(2) or 2x

Think...Write...Pair...Share

18. Correct the error
There is an error in the student work shown below.

Directions: Simplify $\log_2\left(\frac{7x}{3}\right)$.

$$\log_2\left(\frac{7x}{3}\right) = \log_2(7x) - \log_2 3$$
$$= \log_2 7 - \log_2 x - \log_2 3$$

What is the error in the work above?

_____

What should the student have done instead of what he or she did?

_____

*If you finish early, try this one!

18. Explain why $\log_3(x^2 + 2xy + y^2) = 2 \cdot \log_3 (x + y)$.

_____

_____

Think...Write...Pair...Share

18. Correct the error
There is an error in the student work shown below.

Directions: Simplify $\log_2\left(\frac{7x}{3}\right)$.

$$\log_2\left(\frac{7x}{3}\right) = \log_2(7x) - \log_2 3$$
$$= \log_2 7 - \log_2 x - \log_2 3$$

What is the error in the work above?

$\log_2(7x) \neq \log_2 7 - \log_2 x$ like the student said.

What should the student have done instead of what he or she did?

$\log_2(7x) = \log_2 7 + \log_2 x$

*If you finish early, try this one!

18. Explain why $\log_3(x^2 + 2xy + y^2) = 2 \cdot \log_3(x + y)$.

We can factor $x^2 + 2xy + y^2$ to get $(x+y)^2$.

So $\log_3(x^2 + 2xy + y^2) = \log_3(x+y)^2$.

The power property allows $\log_3(x+y)^2 = 2 \cdot \log_3(x+y)$.

Therefore, $\log_3(x^2 + 2xy + y^2) = 2 \cdot \log_3(x+y)$.

Practice Problems

Warm Up Exercises I

Try the following exercises on your own.

Use the product rule to rewrite a sum as the logarithm of a single number

1. $\log_4 3 + \log_4 5 =$ _____

3. $\log_9 2 + \log_9 7 =$ _____

2. $\log_9 2 + \log_9 7 =$ _____

4. $\log_{12} 5 + \log_{12} 11 =$ _____

Write the following sums as the logarithm of a single number.

5. $\log_3 2 + \log_3 7 =$ _____

6. $\log_7 2 + \log_7 3 =$ _____

7. $\log_5 6 + \log_5 11 + \log_5 3 =$ _____

8. $\log_5 2 + \log_5 7 + \log_5 3 =$ _____

II. Warm Ups

Try the following exercises on your own.

Use the quotient rule for logarithms to simplify each expression.

1. $\log_2\left(\frac{11}{2}\right) =$ _____

2. $\log_2\left(\frac{9}{5}\right) =$ _____

3. $\log_2\left(\frac{4}{7}\right) =$ _____

4. $\log_2\left(\frac{18}{7}\right) =$ _____

5. $\log_2\left(\frac{16}{5}\right) =$ _____

6. $\log_2\left(\frac{25}{3}\right) =$ _____

III. Practice Problems

Write the following differences as the logarithm of a single number.

7. $\log_3 10 - \log_3 5 =$ _____

8. $\log_7 14 - \log_7 7 =$ _____

9. $\log_5 6 - \log_5 11 =$ _____

10. $\log_3 24 - \log_3 6 =$ _____

11. $\log_3 33 - \log_3 11 =$ _____

12. $\log_5 100 - \log_5 25 =$ _____

Practice Problems

Warm Up Exercises I

Try the following exercises on your own.

Use the product rule to rewrite a sum as the logarithm of a single number

1. $\log_4 3 + \log_4 5 =$ $\log_4 15$

2. $\log_9 2 + \log_9 7 =$ $\log_9 14$

3. $\log_9 2 + \log_9 7 =$ $\log_9 14$

4. $\log_{12} 5 + \log_{12} 11 =$ $\log_{12} 55$

Write the following sums as the logarithm of a single number.

5. $\log_3 2 + \log_3 7 =$ $\log_3 14$

6. $\log_7 2 + \log_7 3 =$ $\log_7 6$

7. $\log_5 6 + \log_5 11 + \log_5 3 =$ $\log_5 198$

8. $\log_5 2 + \log_5 7 + \log_5 3 =$ $\log_5 63$

II. Warm Ups

Try the following exercises on your own.

Use the quotient rule for logarithms to simplify each expression.

1. $\log_2\left(\frac{11}{2}\right) = \log_2 11 - \log_2 2 = \log_2 11 - 1$

2. $\log_2\left(\frac{9}{5}\right) = \log_2 9 - \log_2 5$

3. $\log_2\left(\frac{4}{7}\right) = \log_2 4 - \log_2 7 = 2 - \log_2 7$

4. $\log_2\left(\frac{18}{7}\right) = \log_2 18 - \log_2 7$

5. $\log_2\left(\frac{16}{5}\right) = \log_2 16 - \log_2 5 = 4 - \log_2 5$

6. $\log_2\left(\frac{25}{3}\right) = \log_2 25 - \log_2 3$

III. Practice Problems

Write the following differences as the logarithm of a single number.

7. $\log_3 10 - \log_3 5 =$ $\log_3(2)$

8. $\log_7 14 - \log_7 7 =$ $\log_7(2)$

9. $\log_5 6 - \log_5 11 =$ $\log_5(6/11)$

10. $\log_3 24 - \log_3 6 =$ $\log_3(4)$

11. $\log_3 33 - \log_3 11 =$ $\log_3(3) = 1$

12. $\log_5 100 - \log_5 25 =$ $\log_5(4)$

Practice Problems continued....

II. Warm Up

Try the following exercises on your own.
Use the power rule for logarithms to simplify each expression.

1. $\log_2(5^3) =$ _____

2. $\log_2(4^{10}) =$ _____

3. $\log_2(64^{12}) =$ _____

4. $\log_2(8^7) =$ _____

5. $\log_3(9^4) =$ _____

6. $\log_3(15^{20}) =$ _____

Simplify the following expressions.

17. $\log_b(a^{100}) =$ _____

18. $\log_b(t^{\theta}) =$ _____

19. $\log_b(x^{10}) =$ _____

20. $\log_b(x^3) =$ _____

21. $\log_b(y^x) =$ _____

22. $\log_b(b^5) =$ _____

Practice Problems continued....

II. Warm Up

Try the following exercises on your own.
Use the power rule for logarithms to simplify each expression.

1. $\log_2(5^3) =$ $3 \cdot \log_2 5$

2. $\log_2(4^{10}) =$ $10 \cdot 2 = 20$

3. $\log_2(64^{12}) =$ $12 \cdot \log_2 64 = 12 \cdot 6 = 72$

4. $\log_2(8^7) =$ $7 \cdot 3 = 21$

5. $\log_3(9^4) =$ $4 \cdot \log_3 9 = 4 \cdot 2 = 8$

6. $\log_3(15^{20}) =$ $20 \cdot \log_3 15$

Simplify the following expressions.

17. $\log_b(a^{100}) =$ $100 \cdot \log_b a$

18. $\log_b(t^9) =$ $9 \cdot \log_b t$

19. $\log_b(x^{10}) =$ $10 \cdot \log_b x$

20. $\log_b(x^3) =$ $3 \cdot \log_b x$

21. $\log_b(y^x) =$ $x \cdot \log_b y$

22. $\log_b(b^5) =$ $5 \cdot \log_b b$
$= 5 \cdot 1 = 5$

Score: _____/ 6 Name _____

Exit Ticket

1) Simplify: $\log_2 5 + \log_2 6$

2) Simplify: $\log_2 5 - \log_2 6$

3) Simplify: $\log_2(5^6)$

Score: _____/ 6 Name _____

Exit Ticket

1) Simplify: $\log_3 7 + \log_3 9$

2) Simplify: $\log_3 9 - \log_3 7$

3) Simplify: $\log_3(9^7)$

Score: _____/ 6 Name _____

Exit Ticket

1) Simplify: $\log_2 5 + \log_2 6$ $\log_2(30)$

2) Simplify: $\log_2 5 - \log_2 6$ $\log_2(5/6)$

3) Simplify: $\log_2(5^6)$ $6 \cdot \log_2(5)$

Score: _____/ 6 Name _____

Exit Ticket

1) Simplify: $\log_3 7 + \log_3 9$ $\log_3(63)$

2) Simplify: $\log_3 9 - \log_3 7$ $\log_3(9/7)$

3) Simplify: $\log_3(9^7)$ $7 \cdot \log_3(9)$

HW: "Logs worksheet #2"

1) Write the following in Logarithmic Form

$2^5 = 32$ _____ (2 pt)

2) Write the following in Exponential Form

$\log_4 16 = 2$ _____ (2 pt)

3) Solve the following for *x*. (1 pt each)

a) $2^x = \frac{1}{8}$ b) $x^3 = 27$ c) $9^{\frac{1}{2}} = x$

x = _____ *x* = _____ *x* = _____

4) Solve the following for *x*. (1 pt each)

a) $10^x = 57$ b) $\log_{10}(12{,}345) = x$ c) $\log(0.10) = x$

x = _____ *x* = _____ *x* = _____

5) Solve the following for *x*. (1 pt each)

a) $x = \log_{10}(100)$ b) $x = \log_{11}(121)$ c) $\log_4(64) = x$

x = _____ *x* = _____ *x* = _____

HW: "Logs worksheet #2"

1) Write the following in Logarithmic Form

$2^5 = 32$ $\log_2 32 = 5$ (2 pt)

2) Write the following in Exponential Form

$\log_4 16 = 2$ $4^2 = 16$ (2 pt)

3) Solve the following for x. (1 pt each)

a) $2^x = \frac{1}{8}$ b) $x^3 = 27$ c) $9^{\frac{1}{2}} = x$

$x = -3$ $x = 3$ $x = 3$

4) Solve the following for x. (1 pt each)

a) $10^x = 57$ b) $\log_{10}(12{,}345) = x$ c) $\log(0.10) = x$

$x =$ _____ $x =$ _____ $x = -1$

5) Solve the following for x. (1 pt each)

a) $x = \log_{10}(100)$ b) $x = \log_{11}(121)$ c) $\log_4(64) = x$

$x = 2$ $x = 2$ $x = 3$

HW: "Logs worksheet #2" continued...

I. Simplify the following: (Product &Quotient properties)

1) $\log_2(5) + \log_2(7) =$ _____

2) $\log_3(45) - \log_3(5) =$ _____

3) $\log_7(3x) - \log_7(9x) + \log_7(6y) =$ _____

2) $\log_2(5) + \log_3(45) =$ _____

II. Simplify the following: (Power property)

1) $\log_5 25^4 =$ _____

2) $\log_3 27^{100} =$ _____

3) $\log_4 4^5 =$ _____

4) $\log_{10} 10^2 =$ _____

III. Simplify the following: (*Challenge)

*5) Solve for x: $9^{\log_9 2} = x$ _____

*6) Solve for x: $5^{\log_5 3} = x$ _____

*7) Write as a single log: $5 \cdot \log_2 m + 2 \cdot \log_2 n =$

_____

HW: "Logs worksheet #2" continued...

I. Simplify the following: (Product &Quotient properties)

1) $\log_2(5) + \log_2(7) =$ $\log_2 35$

2) $\log_3(45) - \log_3(5) =$ $\log_3 9 = 2$

3) $\log_7(3x) - \log_7(9x) + \log_7(6y) =$ $\log_7 2y$

2) $\log_2(5) + \log_3(45) =$ $\log_2 5 + \log_3 45$

II. Simplify the following: (Power property)

1) $\log_5 25^4 =$ $4 \cdot 2 = 8$

2) $\log_3 27^{100} =$ $100 \cdot 3 = 300$

3) $\log_4 4^5 =$ $5 \cdot 1 = 5$

4) $\log_{10} 10^2 =$ $2 \cdot 1 = 2$

III. Simplify the following: (*Challenge)

*5) Solve for x: $9^{\log_9 2} = x$ $\log_9 x = \log_9 2 \rightarrow x = 2$

*6) Solve for x: $5^{\log_5 3} = x$ $\log_5 x = \log_5 3 \rightarrow x = 3$

*7) Write as a single log: $5 \cdot \log_2 m + 2 \cdot \log_2 n =$

$\log_2 m^5 + \log_2 n^2 = \log_2 (m^5 \cdot n^2)$