Lesson 25: Congruence Criteria for Triangles—SAA and HL

Student Outcomes

- Students learn why any two triangles that satisfy the SAA or HL congruence criteria must be congruent.
- Students learn why any two triangles that meet the AAA or SSA criteria are not necessarily congruent.

Classwork

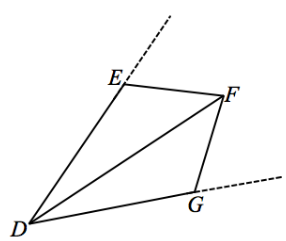
Opening Exercise (7 minutes)

Opening Exercise

Write a proof for the following question. Once done, compare your proof with a neighbor's.

Given: $DE = DG$, $EF = GF$

Prove: DF is the angle bisector of $\angle EDG$



Proof:

$DE = DG$	*Given*
$EF = GF$	*Given*
$DF = DF$	*Common side*
$\triangle DEF = \triangle DGF$	*SSS*
$\angle EDF = \angle GDF$	*Corr. $\angle$s of $\cong$ $\triangle$s*
DF is the angle bisector of $\angle EDG$	*Defn. of angle bisector*

Discussion (25 minutes)

Discussion

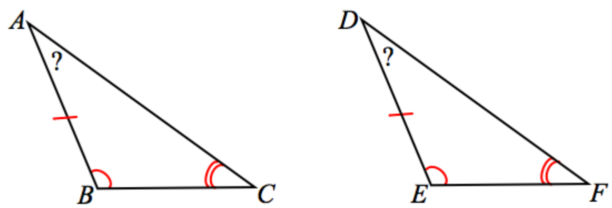
Today we are going to examine three possible triangle congruence criteria, Side-Angle-Angle (SAA) and Side-Side-Angle (SSA), and Angle-Angle-Angle (AAA). Ultimately, only one of the three possible criteria will actually ensure congruence.

Side-Angle-Angle triangle congruence criteria (SAA): Given two triangles ABC and $A'B'C'$. If $AB = A'B'$ (Side), $\angle B = \angle B'$ (Angle), and $\angle C = \angle C'$ (Angle), then the triangles are congruent.

Proof

Consider a pair of triangles that meet the SAA criteria. If you knew that two angles of one triangle corresponded to and were equal in measure to two angles of the other triangle, what conclusions can you draw about the third angles of each triangle?

Since the first two angles are equal in measure, the third angles must also be equal in measure.



Given this conclusion, which formerly learned triangle congruence criteria can we use to determine if the pair of triangles are congruent?

ASA

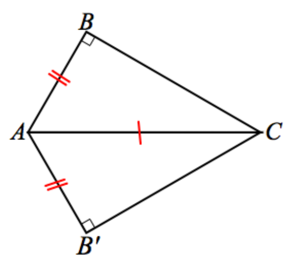
Therefore, the SAA criterion is actually an extension of the ______________ triangle congruence criterion.

ASA

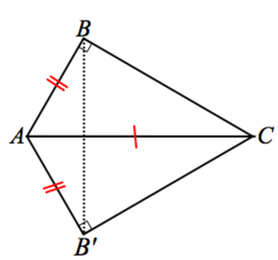
Hypotenuse-Leg triangle congruence criteria (HL): Given two right triangles ABC and $A'B'C'$with right angles $\angle B$ and $\angle B'$. If $AB = A'B'$ (Leg) and $AC = A'C'$ (Hypotenuse), then the triangles are congruent.

Proof

As with some of our other proofs, we will not start at the very beginning, but imagine that a congruence exists so that triangles have been brought together such that $A = A'$ and $C = C'$; the hypotenuse acts as a common side to the transformed triangles.



Similar to the proof for SSS, we add a construction and draw BB'.

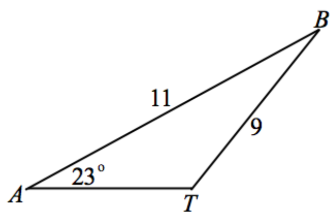


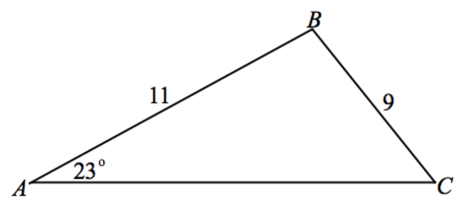
$\triangle ABB'$ is isosceles by definition, and we can conclude that base angles $\angle ABB' = \angle AB'B$. Since $\angle CBB'$ and $\angle CB'B$ are both the complements of equal angle measures ($\angle ABB'$ and $\angle AB'B$), they too are equal in measure. Furthermore, since $\angle CBB' = \angle CB'B$, the sides of $\triangle CBB'$ opposite them are equal in measure $BC = B'C'$.

Then, by SSS, we can conclude $\triangle ABC \cong \triangle A'B'C'$.

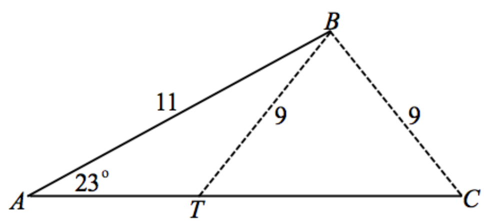
Criteria that *do not* determine two triangles as congruent: SSA and AAA

Side-Side-Angle (SSA): Observe the diagrams below. Each triangle has a set of adjacent sides of measures 11 and 9, as well as the non-included angle of 23˚. Yet, the triangles are not congruent.





Examine the composite made of both triangles. The sides of lengths 9 each have been dashed to show their possible locations.



The pattern of SSA cannot *guarantee* congruence criteria. In other words, two triangles under SSA criteria might be congruent, but they might not be; therefore we cannot categorize SSA as congruence criterion.

Angle-Angle-Angle (AAA): A correspondence exists between triangles $\triangle ABC$ and $\triangle DEF$. Trace $\triangle ABC$ onto patty paper and line up corresponding vertices.

Based on your observations, why can't we categorize AAA as congruence criteria? Is there any situation in which AAA does guarantee congruence?

Even though the angle measures may be the same, the sides can be proportionally larger; you can have similar triangles in addition to a congruent triangle.

List all the triangle congruence criteria here:

SSS, SAS, ASA, SAA, HL

List the criteria that do not determine congruence here:

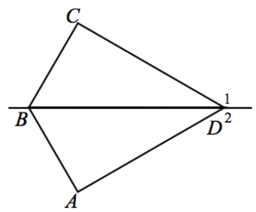
SSA, AAA

Examples (8 minutes)

Examples

1. **Given:** $BC \perp CD$, $AB \perp AD$, $\angle 1 = \angle 2$

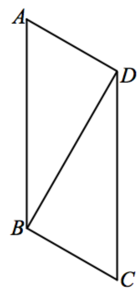
   Prove: $\triangle BCD \cong \triangle BAD$



$\angle 1 = \angle 2$	*Given*
$AB \perp AD$	*Given*
$BC \perp CD$	*Given*
$BD = BD$	*Common Side*
$\angle 1 + \angle CDB = 180°$	*Straight Angle*
$\angle 2 + \angle ADB = 180°$	*Straight Angle*
$\angle CDB \cong \angle ADB$	*Substitution*

2. **Given:** $AD \perp BD$, $BD \perp BC$, $AB = CD$

   Prove: $\triangle ABD \cong \triangle CDB$



$AD \perp BD$	*Given*
$BD \perp BC$	*Given*
$AB = CD$	*Given*
$BD = BD$	*Common Side*
$\triangle ABD \cong \triangle CDB$	*HL*

Exit Ticket (5 minutes)

Name ____ Date____________________

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Exit Ticket

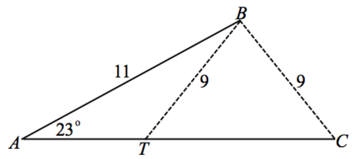
1. Sketch an example of two triangles that meet the AAA criteria but are not congruent.

2. Sketch an example of two triangles that meet the SSA criteria that are not congruent.

Exit Ticket Sample Solutions

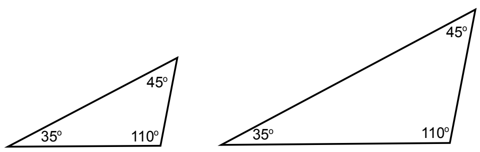
1. **Sketch an example of two triangles that meet the AAA criteria but are not congruent.**

   Responses should look something like the example below.



2. **Sketch an example of two triangles that meet the SSA criteria that are not congruent.**

   Responses should look something like the example below.



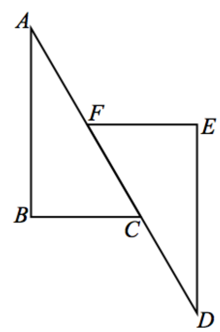
Problem Set Sample Solutions

Use your knowledge of triangle congruence criteria to write proofs for each of the following problems.

1. **Given:** $AB \perp BC,\ DE \perp EF,\ BC \parallel EF,\ AF = DC$

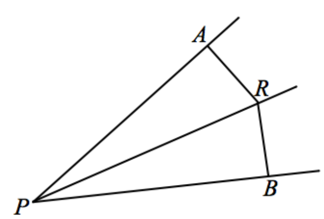
   Prove: $\triangle ABC \cong DEF$

$AB \perp BC$	*Given*
$DE \perp EF$	*Given*
$BC \parallel EF$	*Given*
$AF = DC$	*Given*
$\angle C = \angle F$	*Alt. Int. $\angle$s*
$FC = FC$	*Common Side*
$AF + FC = FC + CD$	*Substitution*
$\Delta ABC \cong \Delta DEF$	*SAA*

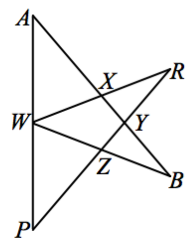


2. **In the figure, $PA \perp AR$ and $PB \perp BR$ and R is equidistant from the lines PA and PB. Prove that PR bisects $\angle APB$.**

$PA \perp AR$	*Given*
$PB \perp BR$	*Given*
$RA = RB$	*Given*
$PR = PR$	*Common Side*
$\Delta PAR \cong \Delta PBR$	*HL*
$\angle APR \cong \angle RPB$	*Corr. $\angle$s of $\cong \Delta$*

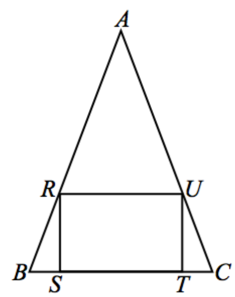


3. **Given:** $\angle A = \angle P$, $\angle B = \angle R$, W is the midpoint of AP
Prove: $RW = BW$



$\angle A = \angle P$	*Given*
$\angle B = \angle R$	*Given*
W is the midpoint of AP	*Given*
$AW = PW$	*Def. of midpoint*
$\Delta AWB \cong \Delta PWR$	*SAA*
$RW = BW$	*Corr. Sides of $\cong$ Δs*

4. **Given:** $BR = CU$, rectangle $RSTU$
Prove: $\triangle ARU$ is isosceles



$BR = CU$	*Given*
rectangle $RSTU$	*Given*
BC$\parallel$ RU	*Def. of Rect.*
$\angle RBS = \angle ARU$	*Corr. $\angle$s*
$\angle UCT = \angle AUR$	*Corr. $\angle$s*
$\angle RST = 90°, \angle UTS = 90°$	*Def. of a Rect.*
$\angle RSB + \angle RST = 180$	*Straight Angle*
$\angle UTC + \angle UTS = 180$	*Straight Angle*
$RS = UT$	*Def. of Rect.*
$\Delta BRS \cong \Delta TUC$	*HL*
$\angle RBS = \angle UCT$	*Corr. $\angle$s of $\cong$ Δs*
$\angle ARU = \angle AUR$	*Substitution*
$\triangle$ *ARU is isosceles*	*Base $\angle$s converse*