Exponential Functions

Objective: SWBAT

- Classify an exponential function as representing exponential growth or exponential decay
- Determine the multiplier for exponential growth and decay
- Write and evaluate exponential expressions to model growth and decay situations

Language Objective: SWBAT use a table to list the characteristics of exponential functions established with different exponents and basis

Agenda

1) Do Now - Sketch exponential graphs

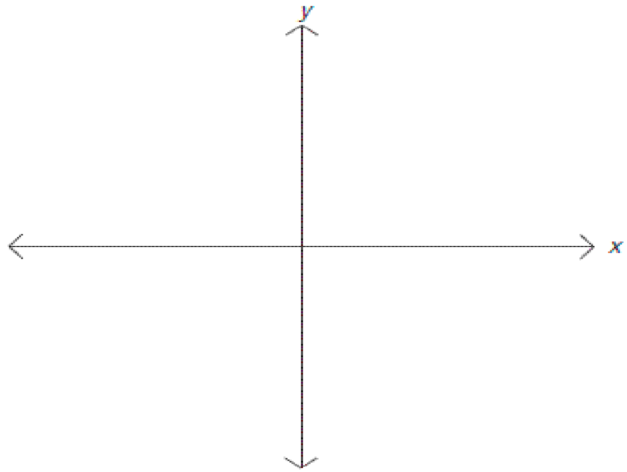
2) Notes - Effects of Exponential Functions
   -Growth and Decay

3) Practice Problems- Together

HW: Worksheet #1

Do Now: **Sketch each function below. Indicate the y-intercept.**

$y = 2^x$ $y = 5^x$ $y = (3)^{-x}$ $y = (1/2)^x$



For all the functions above, state:

the Domain ____________ the Range ____________ the Asymptote ____________

Exponential Function

$$f(x) = b^x$$

Properties of Exponential Functions/Graphs

	x is "+"	x is "–"
$b > 1$		
$0 < b < 1$		

Why **can't** b be 1?

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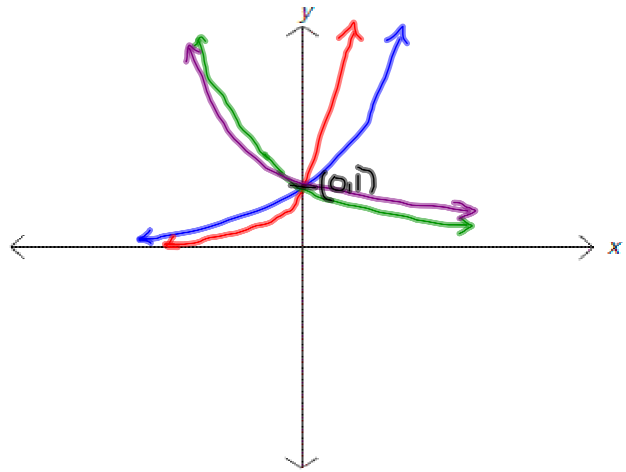


For all the functions above, state:

The Domain x = ℝ The Range y > 0

The Asymptote y = 0

Exponential Function

$$f(x) = b^x$$

Properties of Exponential Functions/Graphs

	x is "+"	x is "–"
$b > 1$		
$0 < b < 1$		

Why **can't** b be 1?

Exponential Function

$$f(x) = b^x$$

x ← power/exponet.
b → any real #
base
→ positive real # except

1^2 1^{-3} $1^{4.5} = 1$

Properties of Exponential Functions/Graphs

	x is "+"	x is "–"
$b > 1$	increasing (growth) the larger the b value, the steeper it is (closer to the y-axis)	decay.
$0 < b < 1$	decreasing (decay) the smaller the b value the steeper it is (closer to y-axis	growth

Why **can't** b be 1? Constant function

$y = 1^x$ $y = b^x$ $b = 1$? $y = 1^5$ $y = 1^{101}$

What coordinate do all exponential functions have in common? (0,1)

Growth and Decay Applications

growth rate: percentage

$f(x) = (\text{initial})(1 \pm r)^x$

Growth

Decay

Find the correct exponential base (the multiplier you will use) for each rate below.

1. 13% growth ________
2. 75% decay ________
3. 8.5% decay ________
4. 7.2% growth ________

Ex. 1

The population of Tokyo-Yokohama, Japan, was about 28, 447, 000 in 1995, and was projected to grow at an annual rate of 1.1%. Predict the population, to the nearest thousand, for the year 2014.

initial population: growth rate: base:

exponent: equation: $f(x) = (\text{initial})(b)^x$

Answer:

Growth and Decay Applications

growth rate: percentage $f(x) = (\text{initial})(1 \pm r)^x$

Growth
add 100% to % given ***(1+r)***
change % to decimal (divide bt 100)
ex) grow by 38%
base : 138%--> b =1.38

Decay
100% subtract % ***(1 - r)***
change to decimal

Find the correct exponential base (the multiplier you will use) for each rate below.

1. 13% growth ***b =1.13***
2. 75% decay ***b =0.25***
3. 8.5% decay ***b=0.915***
4. 7.2% growth ***b =1.072***

Ex. 1

The population of Tokyo-Yokohama, Japan, was about 28, 447, 000 in 1995, and was projected to grow at an annual rate of 1.1%. Predict the population, to the **nearest thousand**, for the year 2014.

initial population: ***28,447,000***

growth rate: ***1.1%***

base: ***b=1.011***

exponent:

2014 - 1995=
= 19

equation: $f(x) = (\text{initial})(b)^x$

$f(x) = 28{,}447{,}000(1.011)^{19}$

Answer:

35,019,378.075... ***= 35,019,000 people!***

Ex. 2

A certain medication is eliminated from the bloodstream at a rate of about 12% per hour. The medication reaches a peak level in the bloodstream of 40 milligrams. Predict the amount, to the nearest tenth of a milligram, of the medication remaining 2 hours after the peak level and 3 hours after the peak level.

initial population: growth rate: base:

exponent (**n**): equation:

Answer:

Ex. 2

A certain medication is eliminated from the bloodstream at a rate of about 12% per hour. The medication reaches a peak level in the bloodstream of 40 milligrams. Predict the amount, to the nearest tenth of a milligram, of the medication remaining 2 hours after the peak level and 3 hours after the peak level.

initial population:
40 mg

growth rate:
-12%

base:
0.88

exponent (**n**):
2
3

equation:
$(40)(0.88)^2$
$(40)(0.88)^3$

Answer:
30.98 mg
27.26 mg

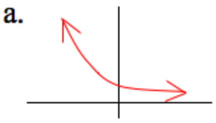
n = # years → time period

f(x) = initial amount (base)$^{\text{time period}}$

Score: _____/6 Name __________________

Exit Ticket

Determine if growth or decay.

1. $y = 100(1.02)^x$	2. $y = 1.02(100)^{-x}$
3. $y = 5(\frac{2}{3})^x$	4. $y = 50(.84)^x$
5. $y = 3(\frac{1}{3})^{-x}$	6. a.  b. 

Score: _____/6 Name __________________

Exit Ticket

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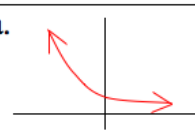
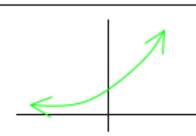
5. $y = 3(\frac{1}{3})^{-x}$	6. a.  b. 

Day 2 Homework:

WORKSHEET #1

Growth and decay *RATES* **Name** ______________

Growth and decay rates are percents and are used to find the base in an exponential equation that uses percents to describe increase or decrease.

For growth:

Your numbers are getting bigger.
You need a multiplier greater than 1 whole.
For a growth rate of 5%, the growth rate **multiplier**
(the base in your exponential equation) is 100% + 5% = 105%
Since percents must be written as decimals (or fractions)
when doing arithmetic, 105% becomes 1.05

For decay:

Your numbers are getting smaller.
You need a multiplier between 0 and 1.
For a decay of 3%, the decay rate **multiplier** is found
by calculating 100% - 3% = 97% which is the decimal .97

NOTE: the multiplier is not the percent given in the problem. You must add that percentage to 100% for growth and subtract it from 100% for decay!

Find the correct exponential base (the multiplier you will use) for each rate below.

1. 1% growth __________ 2. 1.5% decay __________

Use the information above and the exponential model from class notes to solve the following word problems.

3. A new school district is growing at the rate of 4.5% a year. The school presently has a population of 5600 students. If the school continues to grow at 4.5% a year, how many students will it have in 5 years?

a) Initial population __________ b) n = ______ c) decay or growth? _____

d) rate ____________ e) the multiplier (base) =_____

f) equation is ____________ g) Population after 5 years = __________________

Homework:

WORKSHEET #1

Growth and decay *RATES* **Name** ______________

Growth and decay rates are percents and are used to find the base in an exponential equation that uses percents to describe increase or decrease.

For growth:

Your numbers are getting bigger.
You need a multiplier greater than 1 whole.
For a growth rate of 5%, the growth rate **multiplier** (the base in your exponential equation) is 100% + 5% = 105%
Since percents must be written as decimals (or fractions) when doing arithmetic, 105% becomes 1.05

For decay:

Your numbers are getting smaller.
You need a multiplier between 0 and 1.
For a decay of 3%, the decay rate **multiplier** is found by calculating 100% - 3% = 97% which is the decimal .97

NOTE: the multiplier is not the percent given in the problem. You must add that percentage to 100% for growth and subtract it from 100% for decay!

Find the correct exponential base (the multiplier you will use) for each rate below.

1. 1% growth *1.01*

2. 1.5% decay *0.985*

Use the information above and the exponential model from class notes to solve the following word problems.

3. A new school district is growing at the rate of 4.5% a year. The school presently has a population of 5600 students. If the school continues to grow at 4.5% a year, how many students will it have in 5 years?

a) Initial population *5,600* b) n = *5* c) decay or growth? *growth*

d) rate *4.5%* e) the multiplier (base) = *1.045*

f) equation is $(5,600)(1.045)^5$ g) Population after 5 years = *6,978.618*
6,979 students

Day 2 Homework continued...

4. The number of non-violent crimes in Pleasantville has been dropping at the rate of 6% a year. This year the number was 1560. If the drop rate continues to be 6%, how many crimes will there be in 6 years?

a) initial value = ________________ b) *n* = _____ c) growth or decay?

d) rate ________ e) multiplier = _______

d) equation is ________________________________

e) solution = _________________

5. After one hour, a new time -release medication decays in your body at the rate of 25% per hour. If you take a 100mg tablet, model the amount of medication is in your body over time (that means find the equation), and then find out how much medication is in your body after 12 hours.

Identify what each part of the equation represents and write the equation as well as the answer.

Suppose the problem had been worded this way: only 75% of the new medication remains in your body. Is the multiplier (the equation base) .75?

6. One of the radioactive elements in a nuclear meltdown has a half-life of 100 years. This means that the element decays (disappears) at the rate of 50% every 100 years. Until the element decays completely (or to some small pre-determined level), the site is contaminated. Suppose there are 20 grams of element to start. How many grams will there be after 1000 years?

a) Initial value = __________ b) rate ______ c) multiplier = ________

d) *n* = ___________ e) Why is *n* = 10?

f) the equation is ___________________________

g) After 1000 years (10 centuries), there _________ grams left.

Homework continued...

4. The number of non-violent crimes in Pleasantville has been dropping at the rate of 6% a year. This year the number was 1560. If the drop rate continues to be 6%, how many crimes will there be in 6 years?

a) initial value = 1,560 b) n = 6 c) growth or decay? decay

d) rate 6% e) multiplier = 1.06

d) equation is $(1{,}560)(1.06)^6$

e) solution = 2212.88 = **2,213 crimes**

5. After one hour, a new time -release medication decays in your body at the rate of 25% per hour. If you take a 100mg tablet, model the amount of medication is in your body over time (that means find the equation), and then find out how much medication is in your body after 12 hours.

Identify what each part of the equation represents and write the equation as well as the answer.

$(100)(0.75)^{12}$ =

= **3.17 mg**

Suppose the problem had been worded this way: only 75% of the new medication remains in your body. Is the multiplier (the equation base) .75?

yes. b = 0.75

6. One of the radioactive elements in a nuclear meltdown has a half-life of 100 years. This means that the element decays (disappears) at the rate of 50% every 100 years. Until the element decays completely (or to some small pre-determined level), the site is contaminated. Suppose there are 20 grams of element to start. How many grams will there be after 1000 years?

a) Initial value = 20 b) rate 50% c) multiplier = 0.50

d) n = 10 e) Why is n = 10? There are **10 sets** of 100 years in 1000 years

f) the equation is $(10)(0.50)^{10}$

g) After 1000 years (10 centuries), there 0.0098 grams left.