

## COMPEX NUMBERS

### Agenda:

Objective: Evaluate powers of  $i$

(1 presentations for Extra Credit?)

- 1) DO NOW- what is  $i$ ? what is  $i^2$ ?
- 2) Simplify Radicals (#1, 2, 3)
- 3) Exploration of Powers of " $i$ " (#4, 5)
- 4) Summarize Rule - Evaluating Powers of  $i$
- 5) Exit Ticket

HW: "HW Worksheet #1"

## DO NOW

The number  $\sqrt{-1}$  is defined by the letter  $i$

By assigning  $\sqrt{-1} = i$  we are able to continue solving equations we were not previously able to do.

If:  $i = \sqrt{-1}$

Then what is:  $i^2 =$

*When working with a negative number under a radical sign, **deal with "i" first.***

*Therefore:*

$$\sqrt{-4} = \sqrt{-1} \cdot \sqrt{4} = i \cdot \sqrt{4} = i \cdot 2 = 2i$$

*and*

$$\sqrt{-8} = \sqrt{-1} \cdot \sqrt{8} = i \cdot 2\sqrt{2} = 2i\sqrt{2}$$

- Write the following in simplified radical form and for these four problems, show the steps—do not do all work in your head yet!

a.  $\sqrt{-9} =$  \_\_\_\_\_    b.  $\sqrt{-20} =$  \_\_\_\_\_    c.  $3\sqrt{-16} =$  \_\_\_\_\_    d.  $5\sqrt{-18} =$  \_\_\_\_\_

2. Try this:  $\sqrt{-4} \cdot \sqrt{-9} =$  \_\_\_\_\_

(If you did this correctly, the answer is -6.)

If you did not get -6 as your answer, copy the board work.

3. Try to simplify the following problems. (Show your work.)

a.  $\sqrt{-25} \cdot \sqrt{-16} =$  \_\_\_\_\_    b.  $\sqrt{-6} \cdot \sqrt{-2} =$  \_\_\_\_\_    c.  $\sqrt{-36} \cdot \sqrt{4} =$  \_\_\_\_\_

- Correct your work if you missed any of these. You will need the examples later.

4. An exploration of powers of  $i$ . (Remember  $i$  is defined by  $i = \sqrt{-1}$ .)

Directions: Calculate the powers of  $i$ . Try to discover a pattern.

$$i^1 = i \quad i^2 = -1 \quad i^3 = \underline{\hspace{1cm}} \quad i^4 = \underline{\hspace{1cm}}$$

$$i^5 = \underline{\hspace{1cm}} \quad i^6 = \underline{\hspace{1cm}} \quad i^7 = \underline{\hspace{1cm}} \quad i^8 = \underline{\hspace{1cm}}$$

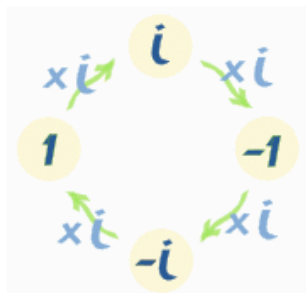
$$i^9 = \underline{\hspace{1cm}} \quad i^{10} = \underline{\hspace{1cm}} \quad i^{11} = \underline{\hspace{1cm}} \quad i^{12} = \underline{\hspace{1cm}}$$

$$i^{13} = \underline{\hspace{1cm}} \quad i^{14} = \underline{\hspace{1cm}} \quad i^{15} = \underline{\hspace{1cm}} \quad i^{16} = \underline{\hspace{1cm}}$$

5. What kind of **pattern** appears to be unfolding? Can you write a **rule** that will help you simplify any power of  $i$ ?

## Interesting Property

The Unit Imaginary Number,  $i$ , has an interesting property. It "cycles" through 4 different values each time you multiply:



- $i \times i = -1$ ,
- then  $-1 \times i = -i$ ,
- then  $-i \times i = 1$ ,
- then  $1 \times i = i$  (back to  $i$  again!)

### Simplifying $i^n$

Step 1. Divide  $n$  by 4 and find the remainder  $r$ .

Step 2. Replace the exponent (power) on  $i$  by the remainder,  $i^n = i^r$ .

Step 3. Use the results  $i^0 = 1$ ,  $i^1 = i$ ,  $i^2 = -1$ , and  $i^3 = -i$  to simplify if necessary.

Solve the following problems and simplify the answers.

1.

Simplify:  
 $i^{35}$



Choose:

- 1  
  $i$   
  $-i$

Explanation

6. Check out the accuracy and usefulness of the rule and try to simplify the following:

$$i^{18} = \underline{\hspace{2cm}} \quad i^{21} = \underline{\hspace{2cm}} \quad i^{40} = \underline{\hspace{2cm}} \quad i^{55} = \underline{\hspace{2cm}}$$

Name \_\_\_\_\_

Score: \_\_\_\_\_/4

Block \_\_\_\_\_

## Exit Ticket

Simplify the following. Use the simplest power of  $i$ .  
Show all work to receive partial credit.

1)  $\sqrt{-49} =$

2)  $i^{62} =$

(Circle the correct answer below)

a)  $i$       b)  $-1$       c)  $-i$       d)  $1$ 

Name \_\_\_\_\_

Score: \_\_\_\_\_/4

Block \_\_\_\_\_

## Exit Ticket

Simplify the following. Use the simplest power of  $i$ .  
Show all work to receive partial credit.

1)  $\sqrt{-81} =$

2)  $i^{27} =$

(Circle the correct answer below)

a)  $i$       b)  $-1$       c)  $-i$       d)  $1$

## 518 Complex Numbers

## HW Worksheet #1

Name \_\_\_\_\_

Block \_\_\_\_\_

1.  $\sqrt{-100} =$

2.  $\sqrt{-40} =$

3.  $\sqrt{-36}$

4.  $-\sqrt{64}$

5.  $\sqrt{-50}$

6.  $\sqrt{-4} \cdot \sqrt{-49} =$

7.  $\sqrt{-3} \cdot \sqrt{-8} =$

8.  $\sqrt{-3} \cdot \sqrt{8}$

9.  $3 \cdot \sqrt{-32} =$

10.  $i^{51} =$

11.  $i^{80} =$

12.  $i^{14} =$

13.  $i^{16} =$

14.  $i^{401}$

15.  $i^0 =$

16.  $i^7 =$

17.  $i^3$

18.  $i^{62} =$

19.  $i^2 \cdot i^3$

20.  $i^5 \cdot i^7 =$

## 518 Complex Numbers

## HW Worksheet #1

Name \_\_\_\_\_

Block \_\_\_\_\_

1.  $\sqrt{-100} =$

$10i$

2.  $\sqrt{-40} =$

$$\sqrt{-4 \cdot 10}$$

$$= 2i\sqrt{10}$$

3.  $\sqrt{-36}$

$6i$

4.  $-\sqrt{64}$

$-8$

5.  $\sqrt{-50}$

$$\sqrt{-25 \cdot 2}$$

$$= 5i\sqrt{2}$$

6.  $\sqrt{-4} \cdot \sqrt{-49} =$

$$4i \cdot 7i$$

$$= 28i^2$$

$$= -28$$

7.  $\sqrt{-3} \cdot \sqrt{-8} =$

$$i\sqrt{3} \cdot \sqrt{-4 \cdot 2}$$

$$= i\sqrt{3} \cdot 2i\sqrt{2}$$

$$= 2i^2\sqrt{6} = -2\sqrt{6}$$

8.  $\sqrt{-3} \cdot \sqrt{8}$

$$i\sqrt{3} \cdot \sqrt{4 \cdot 2}$$

$$= i\sqrt{3} \cdot 2\sqrt{2}$$

$$= 2i\sqrt{6}$$

9.  $3 \cdot \sqrt{-32} =$

$$3 \cdot \sqrt{16 \cdot 2}$$

$$= 3 \cdot 4i\sqrt{2}$$

$$= 12i\sqrt{2}$$

10.  $i^{51} =$

$$4 \overline{) 51} \begin{array}{r} 12 \text{ r } 3 \\ 4 \\ \hline 11 \\ 8 \\ \hline 3 \end{array} = i^3 = -i$$

11.  $i^{80} =$

$$4 \overline{) 80} \begin{array}{r} 20 \text{ r } 0 \\ 4 \\ \hline 180 \\ 160 \\ \hline 20 \end{array} = i^0 = 1$$

12.  $i^{14} =$

$$4 \overline{) 14} \begin{array}{r} 3 \text{ r } 2 \\ 4 \\ \hline 14 \\ 12 \\ \hline 2 \end{array} = i^2 = -1$$

13.  $i^{16} =$

$$4 \overline{) 16} \begin{array}{r} 4 \text{ r } 0 \\ 4 \\ \hline 16 \\ 16 \\ \hline 0 \end{array} = i^0 = 1$$

14.  $i^{401} =$

$$4 \overline{) 401} \begin{array}{r} 100 \text{ r } 1 \\ 4 \\ \hline 401 \\ 400 \\ \hline 1 \end{array} = i^1 = i$$

15.  $i^0 =$

$1$

16.  $i^7 =$

$$4 \overline{) 7} \begin{array}{r} 1 \text{ r } 3 \\ 4 \\ \hline 7 \\ 4 \\ \hline 3 \end{array} = i^3 = -i$$

17.  $i^3 =$

$-i$

18.  $i^{62} =$

$$4 \overline{) 62} \begin{array}{r} 15 \text{ r } 2 \\ 4 \\ \hline 62 \\ 60 \\ \hline 2 \end{array} = i^2 = -1$$

19.  $i^2 \cdot i^3 = i^5$

$$-1 \cdot -i = i$$

20.  $i^5 \cdot i^7 =$

$$i^{12}$$

$$= i^0 = 1$$